Integration Technique - Harder Powers

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Differential Equations

Subtopics: Global or Absolute Minima and Maxima, Modelling Situations, Integration Technique - Harder Powers, Accumulation of Change, Total Amount, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 4

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

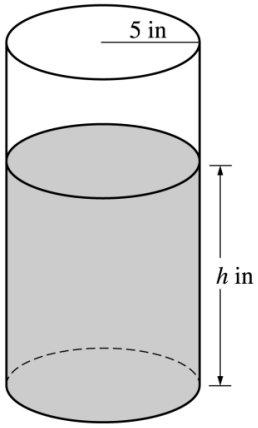
Question 2

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Integration

Subtopics: Related Rates, Particular Solution of Differential Equation, Integration Technique - Harder Powers, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Medium / Question Number: 5



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

Question 3

Qualification: AP Calculus AB

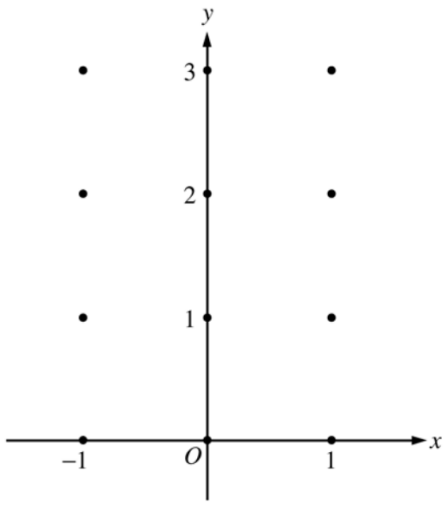
Areas: Differential Equations

Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Medium / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

Question 4

Qualification: AP Calculus AB

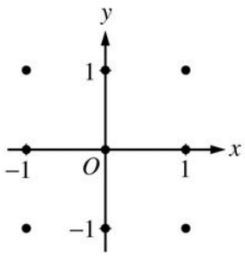
Areas: Differential Equations

Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Integration Technique – Trigonometry, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

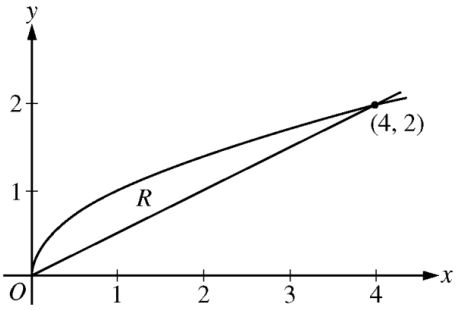
Question 5

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 4



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

Question 6

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Somewhat Challenging / Question Number: 4

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

   (a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

   (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

   (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

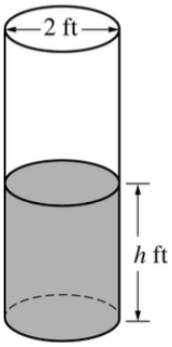
Question 7

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Rates of Change (Instantaneous), Increasing/Decreasing , Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Integration Technique - Harder Powers, Related Rates

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Somewhat Challenging / Question Number: 4



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

   (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

   (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

Question 8

Qualification: AP Calculus AB

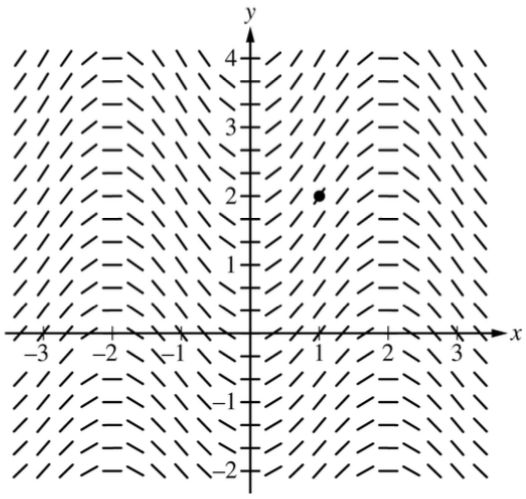
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Trigonometry, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.